

Introducing **floe**: Static and Real-time Calculation of Options Pricing, Flows, and Dealer Gamma, Vanna, and Charm Exposures via Broker Data

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Repository: <https://github.com/FullStackCraft/floe>

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1 Motivation

In recent years, the interest and involvement of retail traders in global markets has surged, including the use of more complex instruments such as options and futures. For the most part, many brokerages have embraced this secular trend by offering their customers a way to stream real-time data directly on their personal computers. However, the standard interface between each brokerage varies widely, and retail traders are left with challenging documentation and inconsistent APIs to work with. We attempt to alleviate this problem by introducing a zero-dependency, browser-only TypeScript library that can compute a variety of both static and real-time options pricing and flow data, including dealer gamma, vanna, and charm exposures from streaming broker data.

2 Introducing `floe`

We introduce `floe`: a zero-dependency, browser-only TypeScript library for computing real-time options flow data, including dealer gamma, vanna, and charm exposures that can be derived purely from real-time streamed broker data alone.

The mathematical foundation of `floe` rests upon the seminal work of Black and Scholes [1] and Merton [2], whose option pricing framework enables the computation of the Greeks used throughout this paper.

3 Paper Structure

The remainder of this paper is divided into two broad sections: the first and main focus of this paper, Section 4 describes our process of calculating in real-time both a smoothed implied volatility surface, the live open interest, and the dealer exposures derived from it. All following sections detail the variety of other options related pricing and analytics that `floe` can compute.

4 Real-time Dealer Exposure Calculations

The real-time dealer exposure calculation consists of two phases: an initialization phase that captures open interest at market open ($t = 0$), and a continuous update phase that recalculates exposures as new spot prices and option quotes arrive. These calculations can be done over a unified interface regardless of the given broker or data source.

4.1 Initialization Phase: Capturing Open Interest

At or before market open, we fetch the complete option chain \mathcal{O} for the underlying symbol. Each option $o \in \mathcal{O}$ contains:

$$o = \langle K, T, \phi, \text{bid}, \text{ask}, \text{OI}_0 \rangle$$

Where:

- K = strike price
- T = expiration timestamp (milliseconds)
- $\phi \in \{\text{call}, \text{put}\}$ = option type
- bid, ask = current bid/ask prices
- OI_0 = open interest at $t = 0$

The market context includes the current spot price S_0 , risk-free rate r , and dividend yield q .

4.2 Implied Volatility Surface Construction

Before calculating the actual dealer exposures, we first need to construct a smooth implied volatility (IV) surface for each expiration T . These per-strike IVs later have a direct affect on the vanna exposure calculation, but also through the gamma and charm calculations themselves via the greeks formulas.

For each option with observed market price P_{mkt} , we solve for σ_{IV} :

$$\text{BS}(S, K, \tau, \sigma_{\text{IV}}, r, q, \phi) = P_{\text{mkt}} \quad (1)$$

Using bisection search with bounds $\sigma \in [0.0001, 5.0]$ (0.01% to 500% volatility).

4.3 Total Variance Smoothing

To ensure arbitrage-free and smooth IV surfaces, we apply total variance smoothing:

1. Convert IV to total variance: $w(K) = \sigma^2 \tau$
2. Apply cubic spline interpolation to $w(K)$
3. Enforce convexity via convex hull projection

4. Convert back to IV: $\sigma_{\text{smooth}}(K) = \sqrt{w(K)/\tau}$

The convexity constraint ensures no calendar spread arbitrage exists in the surface.

4.4 Dealer Position Assumption

We assume dealers are net short options (standard market-maker hedging assumption):

- **Short calls:** Dealers sold calls to retail buyers
- **Long puts:** Dealers bought puts from retail sellers (equivalently, short put exposure is negative)

4.5 Calculate Greeks for Each Option

For each option o with strike K , expiration T , and smoothed IV $\sigma_{\text{IV}}(K)$, we compute the following Greeks using the Black-Scholes-Merton formulas (see Section 5).

We now have all necessary components to compute dealer exposures.

4.6 Exposure Formulas

For each strike K with call open interest OI_C and put open interest OI_P :

Gamma Exposure (GEX):

$$\text{GEX}_K = (-\text{OI}_C \cdot \Gamma_C + \text{OI}_P \cdot \Gamma_P) \cdot (S \cdot 100) \cdot S \cdot 0.01 \quad (2)$$

Because the share multiplier and 1% move resolve to unity, this simplifies to:

$$\text{GEX}_K = (-\text{OI}_C \cdot \Gamma_C + \text{OI}_P \cdot \Gamma_P) \cdot S^2 \quad (3)$$

Where Γ_C and Γ_P are the gamma of the call and put contracts at that expiration and strike, respectively.

Vanna Exposure (VEX):

$$\text{VEX}_K = (-\text{OI}_C \cdot \text{Vanna}_C + \text{OI}_P \cdot \text{Vanna}_P) \cdot (S \cdot 100) \cdot \sigma_{\text{IV}} \cdot 0.01 \quad (4)$$

Reducing constants:

$$\text{VEX}_K = (-\text{OI}_C \cdot \text{Vanna}_C + \text{OI}_P \cdot \text{Vanna}_P) \cdot S \cdot \sigma_{\text{IV}} \quad (5)$$

Where Vanna_C and Vanna_P are the vanna of the call and put contracts at that expiration and strike, respectively, and σ_{IV} is the smoothed implied volatility at that strike.

Charm Exposure (CEX):

$$\text{CEX}_K = (-\text{OI}_C \cdot \text{Charm}_C + \text{OI}_P \cdot \text{Charm}_P) \cdot (S \cdot 100) \cdot 365\tau \quad (6)$$

The factor of 100 accounts for contract multiplier. The 0.01 factor normalizes to a 1% move.

4.7 Total Exposures

Sum across all strikes for each expiration:

$$\text{GEX}_{\text{total}} = \sum_K \text{GEX}_K \quad (7)$$

$$\text{VEX}_{\text{total}} = \sum_K \text{VEX}_K \quad (8)$$

$$\text{CEX}_{\text{total}} = \sum_K \text{CEX}_K \quad (9)$$

$$\text{Net Exposure} = \text{NEX}_{\text{total}} = \text{GEX}_{\text{total}} + \text{VEX}_{\text{total}} + \text{CEX}_{\text{total}} \quad (10)$$

4.8 Real-Time Update Process

The system subscribes to streaming quote data from brokers. On each update event:

Algorithm 1 Real-Time Exposure Update

- 1: **Input:** New quote event (spot price S' or option quote)
 - 2: Update spot price $S \leftarrow S'$
 - 3: Recalculate IV surface Σ for expiration T if option quote received
 - 4: Update live open interest if trade data available
 - 5: **for** each expiration T **do**
 - 6: **for** each strike K **do**
 - 7: $\sigma \leftarrow \text{getIVForStrike}(\Sigma, T, K)$
 - 8: $\tau \leftarrow (T - \text{now})/\text{MS_PER_YEAR}$
 - 9: Compute Γ , Vanna, Charm using updated S, σ, τ
 - 10: Compute $\text{GEX}_K, \text{VEX}_K, \text{CEX}_K$
 - 11: **end for**
 - 12: Aggregate total exposures for expiration T
 - 13: **end for**
 - 14: **Output:** Updated exposure metrics
-

4.9 Live Open Interest Tracking

When intraday trade data is available, we estimate live open interest:

$$\text{OI}_{\text{live}}(t) = \text{OI}_0 + \sum_{i=1}^n \delta_i \quad (11)$$

Where δ_i represents the estimated OI change from trade i , inferred by comparing trade price to NBBO:

- Trade at ask \Rightarrow buyer-initiated \Rightarrow potential OI increase
- Trade at bid \Rightarrow seller-initiated \Rightarrow potential OI decrease or close

4.10 Minimum Broker Requirements

Note for this process to function effectively, brokers must provide a minimum:

- Real-time streaming quotes for underlying and options
- Open interest data before or at market open
- Trade prints with timestamps to estimate live OI changes

`floe` itself could potentially be used to do the rest of all calculations: IV surface construction, Greeks calculation, exposure aggregation, and real-time updates.

5 Black-Scholes Greeks Calculation

For any option, one can compute Greeks with `floe` using the Black-Scholes-Merton model with continuous dividend yield [1, 2].

5.1 Core Parameters

Given spot S , strike K , time to expiry τ (in years), volatility σ , risk-free rate r , and dividend yield q :

$$d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \quad (12)$$

$$d_2 = d_1 - \sigma\sqrt{\tau} \quad (13)$$

5.2 First-Order Greeks

For a **call** option:

$$\Delta_C = e^{-q\tau} N(d_1) \quad (14)$$

$$\Gamma = \frac{e^{-q\tau} n(d_1)}{S\sigma\sqrt{\tau}} \quad (15)$$

$$\Theta_C = -\frac{S\sigma e^{-q\tau} n(d_1)}{2\sqrt{\tau}} - rKe^{-r\tau} N(d_2) + qSe^{-q\tau} N(d_1) \quad (16)$$

$$\mathcal{V} = Se^{-q\tau} \sqrt{\tau} \cdot n(d_1) \quad (17)$$

For a **put** option:

$$\Delta_P = -e^{-q\tau} N(-d_1) \quad (18)$$

$$\Theta_P = -\frac{S\sigma e^{-q\tau} n(d_1)}{2\sqrt{\tau}} + rKe^{-r\tau} N(-d_2) - qSe^{-q\tau} N(-d_1) \quad (19)$$

Where $N(\cdot)$ is the cumulative normal distribution and $n(\cdot)$ is the standard normal PDF:

$$n(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad (20)$$

$$N(x) \approx 1 - n(x) \cdot t \cdot (a_1 + t(a_2 + t(a_3 + t(a_4 + t \cdot a_5)))) \quad \text{for } x > 0 \quad (21)$$

using the Abramowitz-Stegun approximation [4] with $t = 1/(1+0.2316419|x|)$.

5.3 Second-Order Greeks

$$\text{Vanna} = -e^{-q\tau} n(d_1) \frac{d_2}{\sigma} \quad (22)$$

$$\text{Charm}_C = -qe^{-q\tau} N(d_1) - \frac{e^{-q\tau} n(d_1) (2(r-q)\tau - d_2\sigma\sqrt{\tau})}{2\tau\sigma\sqrt{\tau}} \quad (23)$$

$$(24)$$

6 Implied Probability Distribution

`floe` provides functionality to extract the risk-neutral probability distribution implied by option prices, following the Breeden-Litzenberger approach [3].

6.1 Theoretical Foundation

Breeden and Litzenberger demonstrated that the risk-neutral probability density function $f(K)$ of the underlying asset at expiration can be recovered from the second derivative of call option prices with respect to strike:

$$f(K) = e^{r\tau} \frac{\partial^2 C}{\partial K^2} \quad (25)$$

where $C(K)$ is the call option price as a function of strike K , r is the risk-free rate, and τ is time to expiration.

6.2 Numerical Implementation

In practice, we estimate the second derivative using central finite differences on the mid-prices of observed call options:

$$\left. \frac{\partial^2 C}{\partial K^2} \right|_{K_i} \approx \frac{C_{i+1} - 2C_i + C_{i-1}}{(\Delta K)^2} \quad (26)$$

where C_i is the mid-price at strike K_i and $\Delta K = K_{i+1} - K_{i-1}$.

The resulting density values are normalized to sum to unity, yielding a proper probability distribution. From this distribution, we compute summary statistics including the mode (most likely price), median, expected value, and expected move (standard deviation).

7 Shares Needed to Cover

As a sort of global heuristic across all exposures at all strikes for a given expiration, one can estimate the total hedging flow required to neutralize dealer exposure:

$$\text{Shares to Cover} = \frac{-\text{Net Exposure}}{S} \quad (27)$$

$$\text{Implied Move} = \frac{\text{Shares to Cover}}{\text{Shares Outstanding}} \times 100\% \quad (28)$$

The sign indicates directional pressure:

- Negative net exposure \Rightarrow dealers must buy \Rightarrow upward price pressure

- Positive net exposure \Rightarrow dealers must sell \Rightarrow downward price pressure

This method assumes equal weighting of exposure across all strikes, and that the hedge will be perfectly (and exclusively) executed by buying or selling shares of the underlying.

8 Future Work

- Explore concepts of instantaneous local hedge pressure by net exposure at nearest price and how price velocity impacts immediate dealer hedging needs.
- Improve nuances of live open interest estimation by weighing the confidence of each change in OI by how far the trade price is from the mid. Trades clearly at the bid or ask have higher signal; trades near mid price are ambiguous as to aggressor side.

9 Summary

The complete pipeline for real-time dealer exposure calculation:

1. **Initialize:** Fetch option chain with OI_0 at market open
2. **Build IV Surface:** Calculate IV for each option, apply total variance smoothing
3. **Compute Greeks:** For each option using smoothed IV and current spot
4. **Aggregate Exposures:** Calculate GEX, VEX, CEX, and sum them for NEX across strikes per expiration
5. **Stream Updates:** On each new quote, recalculate IV \rightarrow Greeks \rightarrow Exposures
6. **Track Live OI:** Adjust open interest based on observed trades

This methodology enables sub-second exposure updates, providing actionable insight into market-maker hedging dynamics as market conditions evolve.

References

- [1] F. Black and M. Scholes, “The pricing of options and corporate liabilities,” *Journal of Political Economy*, vol. 81, no. 3, pp. 637–654, 1973.

- [2] R. C. Merton, “Theory of rational option pricing,” *Bell Journal of Economics and Management Science*, vol. 4, no. 1, pp. 141–183, 1973.
- [3] D. T. Breeden and R. H. Litzenberger, “Prices of state-contingent claims implicit in option prices,” *Journal of Business*, vol. 51, no. 4, pp. 621–651, 1978.
- [4] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Washington, DC: National Bureau of Standards, Applied Mathematics Series 55, 1964.

A Repository

For the full source, examples, and license, see the project repository:
<https://github.com/FullStackCraft/floe>.